

Strong CP problem, Neutron EDM and Thermal QCD sum rules

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Abstract

The behaviour of the broken CP symmetry at finite temperature is examined. This is achieved through the investigation of the neutron electric dipole moment d_n induced by θ -term. By using thermal QCD sum rules, we find that below the critical temperature, the ratio $|\frac{d_n}{\theta}|$ slightly decreases but survives at temperature effects. This evolution implies that CP remains broken at finite temperature as required by Baryogenesis [1].

1 Introduction

Recently the finite temperature behaviour of symmetries has gained considerable interest. The question of symmetry restoration is a non trivial phenomenon, since it has been shown in [23, 24] that more heat does not necessarily imply more symmetry. Besides, the breaking of the symmetries has a profound implications in a particle physics and cosmology. The CP symmetry is certainly one of the most fundamental symmetries in nature. Besides its role in solving domain wall problem [25], it is a crucial ingredient to understand the matter-antimatter asymmetry [26].

In the three generation Standard Model, CP violation originates from the more obscure sector of the SM: the scalar part. It is parameterized, in the electroweak sector, by a single phase occuring in the Cabbibo-Kobayashi-Maskawa (CKM) quark mixing matrix [2]. CP violation could also originate from additional CP-odd four dimensional operator embedded in the following topological term " θ -term" in the QCD Lagrangian:

$$L_\theta = \theta \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad (1)$$

which breaks P, T and CP. $G_{\mu\nu}$ is the gluonic field strength, $\tilde{G}^{\mu\nu}$ denotes its dual and α_s is the strong coupling constant. The $G_{\mu\nu}\tilde{G}^{\mu\nu}$ quantity is a total derivative which contributes to the physical observables only through non perturbative effects, induced by instantons. A non zero value of θ may generate, in particular, a sizable neutron electric dipole moment (NEDM) which is related to the $\bar{\theta}$ -angle by the following equation obtained within the framework of Chiral perturbation theory:

$$d_n \sim \frac{e}{M_n} \left(\frac{m_q}{M_n} \right) \bar{\theta} \sim \begin{cases} 2.7 \times 10^{-16} \bar{\theta} & [3] \\ 5.2 \times 10^{-16} \bar{\theta} & [4] \end{cases} \quad (2)$$

High precision Experiments have constrained the NEDM to $d_n < 1.1 \times 10^{-25} ecm$ [5], providing a stringent upper limit to $\bar{\theta} < 2 \times 10^{-10}$ [6]. The difficulty to explain the smallness of $\bar{\theta}$ in the standard model is usually known as the "strong CP problem". In this regard, several scenarios were suggested. The most elegant explanation is due to Peccei and Quinn [7], who identified $\bar{\theta}$ to the axion, a very light pseudo scalar boson arising from the spontaneous breaking of a global $U_A(1)$ symmetry. This particle may well be important to explain dark matter puzzle providing a peace of information on the missing mass of the universe [8].

Our aim in this work is to investigate the behaviour of the CP symmetry breaking at finite temperature and the thermal effects on the restoration of the strong CP problem. This is motivated by the possibility to restore some broken symmetries by increasing the temperature.

In section 2, we perform the the calculations of the the $\bar{\theta}$ induced NEDM using thermal QCD sum rules. Section 3 is devoted to the discussion and qualitative analysis of the thermal effects on the CP symmetry restoration.

2 NEDM from thermal QCD sum rules

In order to derive the NEDM through the QCD sum rules techniques [9, 10, 11], we consider a Lagrangian containing the following P and CP violating operators:

$$L_{P,CP} = -\theta_q m_* \sum_f \bar{q}_f i \gamma_5 q_f + \theta \frac{\alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu}. \quad (3)$$

θ_q and θ are respectively two angles coming from the chiral and the topological terms while m_* is the quark reduced mass given by $m_* = \frac{m_u m_d}{m_u + m_d}$. the physical phase is $\bar{\theta} = \theta + \theta_q$. We usually start from the two points correlators in QCD background with a non-vanishing θ in the presence of a constant external electromagnetic field $F^{\mu\nu}$:

$$\Pi(q^2) = i \int d^4x e^{iqx} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle_{\theta, F}. \quad (4)$$

where $\eta(x)$ is the neutron interpolating current [12]:

$$\eta = 2\epsilon_{abc}\{(d_a^T C \gamma_5 u_b)d_c + \beta(d_a^T C u_b)\gamma_5 d_c\}, \quad (5)$$

and β is a mixing parameter. To select the appropriate Lorentz structure, $\Pi(q^2)$ is expanded in terms of the electromagnetic charge as:

$$\Pi(q^2) = \Pi^{(0)}(q^2) + e\Pi^{(1)}(q^2, F^{\mu\nu}) + O(e^2). \quad (6)$$

The first term $\Pi^{(0)}(q^2)$ is the nucleon propagator which includes only the CP-even parameters, while the second term $\Pi^{(1)}(q^2, F^{\mu\nu})$ is the polarization tensor which may be expanded through Wilson OPE as: $\sum C_n <0|\bar{q}\Gamma q|0>_{\theta,F}$, where Γ is an arbitrary Lorentz structure and C_n are the Wilson coefficient functions calculable in perturbation theory [13, 14]. From this expansion, we keep only the CP-odd contribution part. The electromagnetic dependence of these matrix elements is determined in terms of the magnetic susceptibilities κ , χ and ξ [14], defined as:

$$<0|\bar{q}\sigma^{\mu\nu}q|0>_F = \chi e_q F^{\mu\nu} <0|\bar{q}q|0> \quad (7)$$

$$g <0|\bar{q}G^{\mu\nu}q|0>_F = \kappa e_q F^{\mu\nu} <0|\bar{q}q|0> \quad (8)$$

$$2g <0|\bar{q}\tilde{G}^{\mu\nu}q|0>_F = \xi e_q F^{\mu\nu} <0|\bar{q}q|0> \quad (9)$$

Moreover, the θ dependence of $<0|\bar{q}\Gamma q|0>_{\theta}$ matrix elements may be traced by considering the anomalous axial current [10]:

$$m_q <0|\bar{q}\Gamma q|0>_{\theta} = im_*\theta <0|\bar{q}\Gamma q|0> + O(m_q^2) \quad (10)$$

where the correction $O(m_q^2)$ is negligible since $m_{\eta} \gg m_{\pi}$.

Putting altogether the above ingredients and after a straightforward calculation [11], the following expression of $\Pi^{(1)}(q^2, F^{\mu\nu})$ for the neutron is derived:

$$\begin{aligned} \Pi(-q^2) &= -\frac{\bar{\theta}m_*}{64\pi^2} <0|\bar{q}q|0> \{\tilde{F}\sigma, \hat{q}\}[\chi(\beta+1)^2(4e_d - e_u)\ln(\frac{\Lambda^2}{-q^2}) \\ &\quad -4(\beta-1)^2e_d(1 + \frac{1}{4}(2\kappa + \xi))(\ln(\frac{-q^2}{\mu_{IR}^2}) - 1)\frac{1}{-q^2} \\ &\quad -\frac{\xi}{2}((4\beta^2 - 4\beta + 2)e_d + (3\beta^2 + 2\beta + 1)e_u)\frac{1}{-q^2}...], \end{aligned} \quad (11)$$

with $\hat{q} = q_{\mu}\gamma^{\mu}$.

The QCD expression (11) is confronted to the phenomenological parameterization $\Pi^{Phen}(-q^2)$ written in terms of the Neutron hadronic properties. The latter is given by:

$$\Pi^{Phen}(-q^2) = \{\tilde{F}\sigma, \hat{q}\}(\frac{\lambda^2 d_n m_n}{(q^2 - m_n^2)^2} + \frac{A}{(q^2 - m_n^2)} + ...), \quad (12)$$

where m_n is the neutron mass, e_q is the quark charge. the parameters Λ and λ^2 , which originate from the phenomenological side of the sum rule, represent respectively a constant of dimension 2 and the neutron coupling constant to the interpolating current $\eta(x)$. This coupling is defined via a spinor v as $\langle 0|\eta(x)|n \rangle = \lambda v e^{\alpha \gamma_5}$.

In the framework of QCD sum rules, the correlators at finite temperature are expressed in terms of the thermal Gibbs average of Wilson operator expansion [15, 18]. At relatively low temperature, where the system can be regarded as a non interacting gas of bosons, the thermal dependence of the vacuum condensates can be written as :

$$\langle O^i \rangle_T = \langle O^i \rangle + \int \frac{d^3p}{2\epsilon(2\pi)^3} \langle \pi(p) | O^i | \pi(p) \rangle n_B\left(\frac{\epsilon}{T}\right) \quad (13)$$

where $\epsilon = \sqrt{p^2 + m_\pi^2}$, $n_B = \frac{1}{e^{\epsilon/T} - 1}$ is the Bose-Einstein distribution and $\langle O^i \rangle$ is the standard vacuum condensate (i.e. at $T=0$). In this approximation, we only kept the pion contributions, since in the low temperature region, the effects of heavier resonances ($\Gamma = K, \eta, \dots$) are dumped by their distribution functions $\sim e^{-\frac{m_\Gamma}{T}}$ [17]. To compute the pion matrix elements, we apply the soft pion theorem given by:

$$\langle \pi(p) | O^i | \pi(p) \rangle = -\frac{1}{f_\pi^2} \langle 0 | [Q_5^a, [Q_5^a, O^i]] | 0 \rangle + O\left(\frac{m_\pi^2}{\Lambda^2}\right), \quad (14)$$

where Λ is a hadron scale and Q_5^a is the isovector axial charge defined by:

$$Q_5^a = \int d^3x \bar{q}(x) \gamma_0 \gamma_5 \frac{\tau^a}{2} q(x). \quad (15)$$

Direct application of the above formula to the quark and gluon condensates shows the following features [16, 17]:

(i) Only $\langle \bar{q}q \rangle$ is sensitive to temperature. Its behaviour at finite T is given by:

$$\langle \bar{q}q \rangle_T \simeq \left(1 - \frac{\varphi(T)}{8}\right) \langle \bar{q}q \rangle, \quad (16)$$

where $\varphi(T) = \frac{T^2}{f_\pi^2} B\left(\frac{m_\pi}{T}\right)$, $B(z) = \frac{6}{\pi^2} \int_z^\infty dy \frac{\sqrt{y^2 - z^2}}{e^y - 1}$ and f_π is the pion decay constant ($f_\pi \simeq 93 \text{ MeV}$). The variation with temperature of the quark condensate $\langle \bar{q}q \rangle_T$ results in two different asymptotic evolutions, namely:

$$\langle \bar{q}q \rangle_T \simeq \left(1 - \frac{T^2}{8f_\pi^2}\right) \langle \bar{q}q \rangle$$

for $\frac{m_\pi}{T} \ll 1$,

$$\langle \bar{q}q \rangle_T \simeq \left(1 - \frac{\sqrt{\frac{\pi m_\pi}{2T}} T^2}{8f_\pi^2} e^{-\frac{m_\pi}{T}}\right) \langle \bar{q}q \rangle$$

for $\frac{m_\pi}{T} \gg 1$.

(ii) The gluon condensate is nearly constant at low temperature and a T dependence occurs only at order T^8 .

The determination of the ratio $\frac{d_n}{\bar{\theta}}$ sum rules at non zero temperature is now easily performed by applying Borel operator to both parameterizations of the Neutron correlation function shown in (11) and (12). Then finite temperature effects are introduced via the procedure discussed above. Finally, by invoking the quark-hadron duality, we deduce the final sum rules of the $\bar{\theta}$ induced NEDM at finite temperature:

$$\frac{d_n}{\bar{\theta}}(T) = -\frac{M^2 m_*}{16\pi^2} \frac{1}{\lambda_n^2(T) M_n(T)} \left(1 - \frac{\varphi(T)}{8}\right) < \bar{q}q > \left[4\chi(4e_u - e_d) - \frac{\xi}{2M^2}(4e_u + 8e_d)\right] e^{\frac{M_*^2}{M^2}}, \quad (17)$$

where M represents the Borel parameter.

In order to get rid of the infrared divergence, the value of β has been set to 1 in (17). The Thermal evolution of the coupling constant and the mass of the neutron were determined from the thermal nucleon sum rules [17].

Within the dilute pion gas approximation, Eletsky has shown that the contribution induced the pion-nucleon scattering has to be considered [19]. It enters the nucleon sum rules through the coupling constant $g_{\pi NN}$, whose values lie within the range 13.5-14.3 [20].

Numerical analysis is performed with the following input parameters: the Borel mass has been chosen within the values $M^2 = 0.55 - 0.7 GeV^2$ which correspond to the optimal range (Borel window) in the $\frac{d_n}{\bar{\theta}}$ sum rule at $T = 0$ [11]. For the χ and ξ susceptibilities we take $\chi = -5.7 \pm 0.6 GeV^{-2}$ [22] and $\xi = -0.74 \pm 0.2$ [?]. As to the vacuum quark condensate appearing in (17), we use its standard values [9].

3 Analysis and Conclusion

We have established the relation between the NEDM and $\bar{\theta}$ angle at non zero temperature from QCD sum rules. We find that the behaviour of the ratio $\frac{d_n}{\bar{\theta}}$ is connected to the thermal evolution of the pion parameters f_π , m_π and of $g_{\pi NN}$.

By analyzing the ratio as a function of T in the region of validity of thermal sum-rules $[0, T_c]$, we learn that $|\frac{d_n}{\bar{\theta}}|$ decreases smoothly with T (about 16% variation for temperature values up to 200 MeV) but survives at finite temperature. This means that either the NEDM value decreases or $\bar{\theta}$ increases. Consequently, for a fixed value of $\bar{\theta}$ the NEDM decreases but it does not exhibit any critical behaviour. Furthermore, if we start from a non vanishing $\bar{\theta}$ value at $T = 0$, it is not possible to remove it at finite temperature. We also note that $|\frac{d_n}{\bar{\theta}}|$ grows as M^2 or χ susceptibility increases. It also grows with quark condensate rising. However this

ratio is insensitive to both the ξ susceptibility and the coupling constant $g_{\pi NN}$. We notice that for high temperatures, the analysis of $|\frac{d_n}{\theta}| = f(\frac{T}{T_c})$ exhibits a brutal increase justified by the fact that for T beyond the critical value T_c , at which the chiral symmetry is restored, the constants f_π and $g_{\pi NN}$ become zero and consequently the ratio $\frac{d_n}{\theta}$ behaves as a non vanishing constant. The large discrepancy between the values of the ratio for $T < T_c$ and $T > T_c$ may originate from the other contributions to the spectral function which have been neglected, such as the scattering process $N + \pi \rightarrow \Delta$. These contributions, which are of the order T^4 , are negligible in the low temperature region but become substantial for $T \geq T_c$. Moreover, this difference may also be due to the use of soft pion approximation which is valid essentially for low T ($T < T_c$). Therefore it is clear from this qualitative analysis, which is based on the soft pion approximation, that temperature does not play a fundamental role in the suppression of the undesired θ -term and hence the broken CP symmetry is not restored [1]. Indeed, some exact symmetries can be broken by increasing temperature [23, 24]. The symmetry non restoration phenomenon, which means that a broken symmetry at $T=0$ remains broken even at high temperature, is essential for discrete symmetries, CP symmetry in particular. Indeed, the symmetry non restoration is a crucial ingredient in solving the domain wall problem [25] and to create the baryon asymmetry in the early universe (BAU) [26].

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